QUASISTEADY THERMAL REGIME DEVELOPING IN PERIODIC PULSED HEATING OF CYLINDRICAL SOLID BODIES

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An oscillating component of the quasisteady cylindrical temperature field developing in the heating of solids by intense modulated heat sources is investigated.

Recently the creation of new technologies based on modulated laser and electron beams [1-3] has stimulated studies of the interaction of intense periodic pulse heat sources with solids [4-6]. Their urgency is also dictated by the need to use radio-frequency temperature fluctuations to measure the properties of films and thin foils [7, 8] as well to forecast the behavior of construction and device elements under periodic heat loads [9, 10].

The modeling of such an interaction required, as a rule, the solution of a rather complicated nonlinear problem, which can be realized only by using computation methods [4, 5]. However, in the particular, but important case of a quasisteady thermal regime the problem can be linearized to yield an exact analytical solution. For two-dimensional bodies, this solution was obtained in [6-8]. In [7], it was shown that the main difference of periodic pulsed heating from sinusoidal heating is a dependence of the form of the oscillating component of the temperature field Θ on the frequency characteristics of the heating process. In this case, the frequency spectra of an external heat source and the oscillating component have been found to coincide, which was predicted in [6].

For cylindrical solid bodies, the form of temperature fluctuations Θ depends on the pulse repetition frequency ω, as was found experimentally in [11, 12]. However, nobody has succeeded in revealing this dependence analytically. Therefore it is still unclear how the stable forms of temperature fluctuations found for two-dimensional bodies in [7] develop in this case. In this connection, the present work is aimed at constructing a mathematical model of the quasisteady thermal regime that develops in axisymmetrical periodic pulsed heating of solid and hollow cylinders and a body with a cylindrical cavity. The model has been used for the investigation of temperature fluctuations Θ within wide ranges of ω and sample thicknesses.

To linearize the problem, we represent the temperature field T as a sum of the steady T and oscillating Θ components $T(t, r) = \overline{T}(r) + \Theta(t, r)$. At $|\Theta| << T$, the equation for the oscillating component is as follows

$$\frac{\partial \Theta}{\partial t} = a\left(\frac{\partial^2 \Theta}{\partial r^2} + \frac{1}{r}\frac{\partial \Theta}{\partial r}\right) + \mathrm{E}(r); \quad \mathrm{E}(r) = \frac{a}{r}\frac{d}{dr}\left(r\frac{d\overline{T}}{dr}\right). \tag{1}$$

Unlike the one-dimensional case [6], the remainder term $\mathrm{E}(r)$ in the input equation cannot be neglected, even under the assumption that the temperature distribution with respect to the radius is $\overline{T}(r) = \eta_1 r + 1/2\eta_2 r^2$ with small η_1 and η_2. Indeed, for this case, with the exception of more complicated distributions, the requirement

$$\mathrm{E}(r) = a(\eta_1/r + 2\eta_2) << \frac{a}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \Theta}{\partial r}\right)$$

means that $r^{-1}\overline{T}(r)$ must serve as a linear function with small coefficients, i.e., it is required that $\lim_{r \to +0} r^{-1}\overline{T}(r)$ and $r_0^{-1}\overline{T}(r_0)$ to be small. While the first assumption seems reasonable, the second assumption does not follow from the input equation in any way, and it is unclear physically. Therefore, in the present work we examine a general statement of the problem with subsequent evaluation of the remainder terms of its solution.

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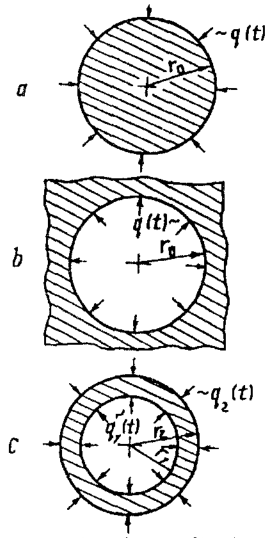

Fig. 1. Periodic pulsed heating of solid cylinder (a), cylindrical cavity (b), and hollow cylinder (c).

Linearization of boundary conditions, as in [6], yields

$$\left(\pm \lambda \frac{\partial \Theta}{\partial r} + \alpha \Theta \right) \Bigg|_{r=r_0} = \bar{q}(t) ; \quad \bar{q}(t) = -\bar{q} + q(t) , \tag{2}$$

$$q(t) = q(t + \nu t_n) , \quad \nu = 1 , 2 \ldots ;$$

$$\bar{q} = \sigma \varepsilon \, [\pm \lambda \bar{T}_r' + (\bar{T}^4 - \bar{T}_0^4)]_{r=r_0} ,$$

where the plus and minus signs stand for a solid cylinder and a body with a cylindrical cavity, respectively; $\bar{q}$ and $q(t)$ are the mean and instantaneous powers released upon heating.

At first we will consider an oscillating regime in the solid cylinder $0 \le r \le r_0$ (Fig. 1a). Having performed the Laplace transformation in (1) and (2), we arrive at

$$\frac{d^2\tilde{\Theta}}{dr^2} + \frac{1}{r} \frac{d\tilde{\Theta}}{dr} + \mu^2 \tilde{\Theta} = \eta(r) ; \quad \tilde{\Theta} = \tilde{\Theta}(p , r) ; \tag{3}$$

$$\left(\lambda \frac{d\tilde{\Theta}}{dr} + \alpha \tilde{\Theta} \right) \Bigg|_{r=r_0} = \tilde{\bar{q}}(p) ; \quad \tilde{\bar{q}}(p) = \frac{\bar{c}_0}{p} + \sum_{\substack{k=-\infty \\ k \neq 0}}^{\infty} \frac{\bar{c}_k}{p - i\omega_k} ;$$

$$\mu = i \sqrt{p/a} ; \quad \eta(r) = -\mathrm{E}(r)/a ; \quad \omega_k = 2\pi k / t_n ,$$

where $\bar{c}_k$ and $\bar{c}_0$ are expansions of $\bar{q}(t)$ into a Fourier series and for periodic pulse loading have the form

$$\bar{c}_k = q_0 \frac{\sin \psi_k}{\pi k} \exp(-i\psi_k) ; \quad \psi_k = \pi k \gamma ; \quad k \neq 0 ;$$

$$\bar{c}_0 = -\bar{q} + c_0 ; \quad c_0 = \gamma q_0 .$$

Here $\gamma = t_u / t_n$ is the pulse ratio, and t_u and t_n are the pulse duration and its repetition frequency, respectively. In this case, the experimentally observed coincidence of the frequency spectra of the external heat source and the oscillatory temperature regime in a quasisteady state results in the relations $\bar{c}_0 = 0$ and $\bar{q} = \gamma q_0$.

A general solution of problem (3) is

$$\tilde{\Theta}(p, r) = \sum_{i=1,2} c_i(p)\,\Gamma_i(\mu r) + \int\limits_0^r \frac{\begin{vmatrix} \Gamma_1(\mu r) & \Gamma_2(\mu r) \\ \Gamma_1(\mu\rho) & \Gamma_2(\mu\rho) \end{vmatrix}}{\begin{vmatrix} \Gamma_1(\mu\rho) & \Gamma_2(\mu\rho) \\ \mu\Gamma_1'(\mu\rho) & \mu\Gamma_2'(\mu\rho) \end{vmatrix}}\,\eta(\rho)\,d\rho, \tag{4}$$

where $\Gamma_i(\mu r) = H_0^{(i)}(\mu r)$ is a zeroth-order Hankel function. Using the Liouville formula, we obtain from (4)

$$\tilde{\Theta}(p, r) = \sum_{i=1,2} c_i(p)\,\Gamma_i(\mu r) + \mu^{-1}\varphi_1(r); \tag{5}$$

$$\varphi_1(r) = \kappa_2(r)\,\Gamma_1(\mu r) - \kappa_1(r)\,\Gamma_2(\mu r);$$

$$\kappa_1(r) = \int\limits_0^r \rho\Gamma_1(\mu\rho)\,\eta(\rho)\,d\rho; \quad \kappa_2(r) = \int\limits_0^r \rho\Gamma_2(\mu\rho)\,\eta(\rho)\,d\rho.$$

The regularity condition at $r \to +0$ yields $c_1 = c_2 = 1/2c$ and

$$\sum_{i=1,2} c_i(p)\,\Gamma_i(\mu r) = c(p)\,J_0(\mu r),$$

while the boundary condition gives an expression for the integration constant $c(p)$, so that

$$\tilde{\Theta}(p, r) = \frac{\tilde{q}(p)\,J_0(\mu r) - \mu^{-1}\varphi_2(r_0)}{\alpha J_0(\mu r_0) - \lambda\mu J_1(\mu r_0)} + \mu^{-1}\varphi_1(r), \tag{6}$$

$$\varphi_2(r_0) = [\lambda\varphi_1(r) + \alpha\varphi_1(r)]_{r=r_0}.$$

The functions $\varphi_{1,2}$ that describe the influence of the remainder term $\mathrm{E}(r)$ can be estimated as $\varphi_{1,2} \sim o(1/\sqrt{p})$, $p \to \infty$. This indicates that the correction introduced by the remainder term into the solution has the form o (1), $t \to \infty$ and $\mathrm{E}(r)$ can be neglected in Eq. (1). Therefore, assuming that the function $\alpha J_0(\mu r_0) - \lambda\mu J_1(\mu r_0)$ in (6) has zeros only in the half-plane Re $p < 0$ and that the path of integration can be transformed as in [6], we obtain

$$\Theta(t, r) = \frac{2\Theta_0}{\pi} \sum_{k=1}^{\infty} \frac{\sin\psi_k}{k}\,\mathrm{Re}\,\frac{J_0(\beta_k r)}{\Delta(r_0)}\,\exp(i(\omega_k t - \psi_k)); \tag{7}$$

$$\Delta(r_0) = J_0(\beta_k r_0) - h_k J_1(\beta_k r_0); \quad \beta_k = i\sqrt{i\omega_k/a}; \quad h_k = \lambda\beta_k/\alpha.$$

Here $J_0(z)$ and $J_1(z)$ are zeroth-order Bessel functions.

For the cylindrical cavity $r_0 \le r < \infty$ (Fig. 1b), the condition at infinity $\Theta(t, \infty) = 0$ in formula (5) yields $c_2(p) = 0$ and $\Theta(p, r) = c(p)\Gamma_1(\beta_k r)$. With the same assumptions made, similar calculations lead to an expression for the oscillating part of the solution for the body with a cylindrical cavity:

$$\Theta(t, r) = \frac{2\Theta_0}{\pi} \sum_{k=1}^{\infty} \frac{\sin\psi_k}{k}\,\mathrm{Re}\,\frac{\Gamma_1(\beta_k r)}{\Delta(r_0)}\,\exp(i(\omega_k t - \psi_k)); \tag{8}$$

$$\Delta(r_0) = \Gamma_1(\beta_k r_0) - h_k\Gamma_1'(\beta_k r_0); \quad \Theta_0 = q_0/\alpha.$$

The most general statement of the problem for periodic pulsed heating of a hollow cylinder $r_1 \le r \le r_2$ (Fig. 1c), in which relations (7) and (8) represent the particular cases, consists in introducing heat sources q_1 and q_2 acting, respectively, on internal and external surfaces of the cylinder. The boundary conditions for Eq. (1) and the time dependence of heat fluxes on the internal $j = 1$ and external $j = 2$ surfaces are of the form

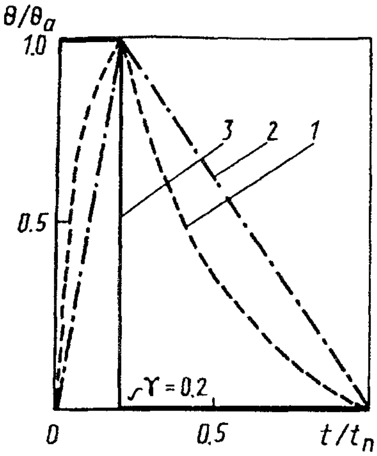

Fig. 2. Form of temperature fluctuations developing with heating of solid and hollow cylinders and a cylindrical cavity.

$$\left[(-1)^j \lambda \frac{\partial \Theta}{\partial r} + \alpha \Theta\right]_{r=r_j} = \bar{q}_j(t); \quad \bar{q}_j(t) = -\bar{q}_j + q_j(t); \tag{9}$$

$$q_j(t) = \begin{cases} q_{j0}; & 0 \le t \le t_{ju}, \quad q_j(t) = q_j(t + \nu t_{jn}), \\ 0; & t_{ju} < t < t_{jn}, \quad \nu = 1, 2, \ldots; \; j = 1, 2. \end{cases}$$

Passing in (9) to the transforms, we obtain a system for determination of the unknown coefficients $c_i = c_i(p)$ of solution (5)

$$\sum_{i=1,2} [(-1)^j i \sqrt{Dp}\, \Gamma_i'(\mu r_j) + \Gamma_i(\mu r_j)]\, C_i = \tilde{\bar{q}}(p)/\alpha; \tag{10}$$

$$\tilde{\bar{q}}_j(p) = \sum_{k \ne 0} \frac{\bar{c}_{jk}}{p - i\omega_{jk}}; \quad \bar{C}_{jk} = (q_{j0} \sin \psi_{jk}/\pi k) \exp(-\psi_{jk});$$

$$\omega_{kj} = 2\pi k/t_{nj}; \quad j = 1, 2; \quad \psi_{jk} = \pi k \gamma_j; \quad \gamma_j = t_{ju}/t_{jn}.$$

Whence

$$\bar{C}_1/\Delta = q_1 [i\sqrt{Dp}\, \Gamma_2(\mu r_2) + \Gamma_2(\mu r_2)] - \tilde{\bar{q}}_2 [-i\sqrt{Dp}\, \Gamma_2(\mu r_1) + \Gamma_2(\mu r_1)]; \tag{11}$$

$$\bar{C}_2/\Delta = -q_1 [i\sqrt{Dp}\, \Gamma_1(\mu r_2) + \Gamma_1(\mu r_2)] + \tilde{\bar{q}}_2 [-i\sqrt{Dp}\, \Gamma_1(\mu r_1) + \Gamma_1(\mu r_1)]. \tag{12}$$

Here $\Delta = \Delta(p)$ is the determinant of system (10).

Formulas (11)-(12) give a solution of the problem in form of a contour integral

$$\Theta(t, r) = \frac{1}{2\pi i} \int_{p_0+i\infty}^{p_0+i\infty} [c_1(p)\, \Gamma_1(i\sqrt{p/a}\, r) + c_2(p)\, \Gamma_2(i\sqrt{p/a}\, r)]\, \Delta^{-1}(p) \exp(pt)\, dp; \quad (p_0 > 0). \tag{13}$$

The expression for the oscillating component of the solution, built from integral representation of (13) with the aid of appropriate deformation of the path of integration $p \in \,]\, p_0 - i\infty, p_0 + i\infty\, [$ in the region $G{:}\,|\arg p| < \pi$ of the complex plane $\mathbb{C}_p$, is as follows:

$$\Theta(t, r) = 2 \sum_{k=1}^{\infty} \sum_{j=1,2} (c_{jk}/\Delta_{jk})\, [S_{jk}^{(1)}\, \Gamma_1(\beta_{jk} r) + S_{jk}^{(2)}\, \Gamma_2(\beta_{jk} r)]; \tag{14}$$

$$S_{1k}^{(1)} = h_{1k}\Gamma_2'(\beta_{1k}r_2) + \Gamma_2(\beta_{1k}r_2); \quad S_{1k}^{(2)} = -h_{1k}\Gamma_1'(\beta_{1k}r_2) - \Gamma_1'(\beta_{1k}r_2);$$

$$S_{2k}^{(1)} = h_{2k}\Gamma_2'(\beta_{2k}r_1) - \Gamma_2(\beta_{2k}r_1); \quad S_{2k}^{(2)} = -h_{2k}\Gamma_1'(\beta_{2k}r_1) + \Gamma_1(\beta_{2k}r_1);$$

$$\Delta_{jk} = \Delta(p_{jk}); \quad h_{jk} = \lambda\beta_{jk}/\alpha; \quad \beta_{jk} = i\sqrt{i\omega_{jk}/a}.$$

Relations (7), (8), and (14) are valid for arbitrary values of the pulse repetition frequency ω and the pulse ratio γ. Since the coefficients $\bar{c}_k$ and $\bar{c}_{ik}$ are introduced into the formulas in the most general form, the relations obtained hold for any piecewise smooth functions $q(t)$ and $q_j(t)$ with period t_m. The case of harmonic heating discussed in [12] for $\alpha = 0$ corresponds to $k = 1$. The case of П-shaped heating (meander) experimentally studied in [11, 12] follows from formula (14), if we assume that $c_{2k} = 0$ and $\gamma_1 = 1/2$.

Figure 2 shows the characteristic forms of temperature fluctuations $\Theta(t)$ calculated by formulas (7), (8), (14) for the surfaces $r = r_0$ (a cylindrical cavity and a solid cylinder) and $r = r_1$ (a hollow cylinder) for different pulse repetition frequencies and sample thickness.

Curves 1-3 pertain to the periodic pulsed thermal regime, the regular thermal regime of the 2nd kind, and the periodic steady thermal regine, respectively. Calculation results are reported for pulse ratio $\varphi = 0.2$. The sample thickness and the pulse repetition frequences for the above curves are related as: $\delta_1 > \delta_2 > \delta_3$; $\omega_1 > \omega_2 > \omega_3$. For definiteness, it is assumed that the heat flux over the external surface of the hollow cylinder is $q_2 = 0$; the material of the sample is tungsten. For clarity, the curves are normalized to unity and the time scale is given in parts of the period t_n. The thermophysical constants [13] are used. The calculations show that, as in the case of two-dimensional samples [6, 7], in periodic pulsed heating of bodies with cylindrical symmetry three quasisteady thermal regimes develop, in which the form of the temperature fluctuations $\Theta(t, r)$ remains constant in a sufficiently wide range of the pulse repetition frequencies and sample thicknesses. In the cylindrical cavity, as in a half-space, only a periodic pulsed heating regime can exist [6] when the temperature fluctuations in the heating stage at $\gamma \le 1/4$ coincide with those calculated by formula [14] for a single pulse (curve 1). In the case of solid and hollow cylinders all three regimes are realized: periodic-pulse [7], regular of the second kind [11, 12] (the pulse shape depends linearly on time, curve 2), and periodic steady regimes [8] (the dependence $\Theta(t, r)$ is a replica of the thermal-pulse shape, curve 3). The sequence of passing from one stable thermal regime to another with changing of the pulse repetition frequency ω and the sample thickness δ is shown in the figure. Taking into account the result obtained for a plate [7], we can assume that stability of the quasisteady thermal regime is determined by $\mathrm{Pd} = \omega\delta^2/a$, where the characteristic size δ (sample thickness) is equal to r_0 for solid and $r_2 - r_1$ for hollow cylinders. For a cylindrical cavity, the characteristic size is infinite and, consequently, $\mathrm{Pd} \to \infty$. In particular, for the cases shown in Fig. 2 $\mathrm{Pd}_1 \ge 10$, $\mathrm{Pd}_2 \sim 10^{-1} - 10^{-2}$, and $\mathrm{Pd}_3 \le 10^{-6}$ correspond to curves 1, 2, and 3, respectively.

Thus, a theoretical study has been made of an oscillating component of the temperature field developing on periodic pulsed heating of bodies of cylindrical configuration. A relationship is found between the form of temperature fluctuation and the body geometry. It is shown that, as in the one-dimensional case, only three quasisteady thermal regimes exist, which do not change with the pulse repetition frequency or the sample thicknesses.

NOTATION

q, specific heat flux; t_u and t_n, pulse duration and repetition frequency; ω, circular frequency of pulse repetition; γ, pulse ratio; $\bar{c}_k$, coefficients of Fourier series expansion of heat flux $\bar{q}(t)$; T, temperature; $\bar{T}$, mean temperature; Θ, oscillating temperature component; $\Gamma_i(z)$, $J_0(z)$, Hankel and Bessel functions of the zeroth order.

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